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Quantum interference in the time-of-flight distribution

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Abstract

We propose a scheme to experimentally observe matter-wave interference in the time domain, specifically in the arrival time or time-of-flight (TOF) distribution for an atomic Bose–Einstein condensate (BEC) Schrödinger-cat state represented by superposition of macroscopically separated wave packets in space. This is in contrast to interference in space at a fixed time observed in the reported BEC experiments. We predict and quantify the quantum interference in the TOF distribution calculated from the modulus of the quantum probability current density (rather than the TOF distributions obtained from a purely classical or semi-classical treatment in many reported experiments). The interference and hence the coherence in the quantum TOF signal disappears in the large-mass limit. Our scheme has the potential to probe the validity of various other theoretical approaches (Muga and Leavens 2000 *Phys. Rep.* **338** 353) of calculating the quantum arrival time distribution.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

In recent times, laser cooling and trapping of atoms has become an area of active research [1]. The temperature of the cold atomic sample is one of its most important characteristics and several methods have been proposed and used for its determination. A well-known technique of measuring this temperature is the time-of-flight (TOF) method [2]. It is significant to mention that the first evidence for a Bose–Einstein condensate (BEC) emerged from TOF measurements [3]. Most of the samples of cold atoms are initially prepared in magneto-optical traps and the atomic cloud is allowed for a thermal expansion after its release from the trap. These so-called TOF measurements are performed either by acquiring the absorption signal of the probe laser beam through the falling and expanding atomic cloud or by measuring the fluorescence of the atoms excited by the resonant probe light. Most of the theoretical analyses

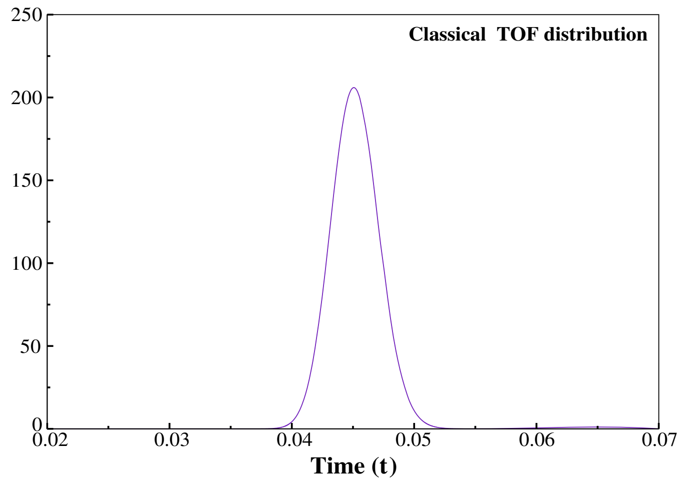

Figure 1. Classical TOF distribution $\mathscr{D}(t)$ given by equation (3) is plotted for BEC of sodium atoms at a temperature $T = 1\,\mu$K. The detector is located at a distance $z = H = -1$ cm and $\sigma_0 = 1\,\mu$m.

of TOF measurements are as follows. To find the shape of the absorption TOF signal, one assumes to start with the initial Gaussian position and velocity distributions of atoms in the trapped sample. The initial probability distribution of finding an atom in the phase space volume element with coordinates (z_0, v_0) is given by

$$\mathscr{D}(z_0, v_0)\,\mathrm{d}z_0\,\mathrm{d}v_0 = \frac{1}{\left(2\pi\sigma_0^2\right)^{1/2}} \exp\left(-\frac{z_0^2}{2\sigma_0^2}\right) \frac{1}{\left(2\pi\sigma_v^2\right)^{1/2}} \exp\left(-\frac{v_0^2}{2\sigma_v^2}\right) \mathrm{d}z_0\,\mathrm{d}v_0. \tag{1}$$

Here for simplicity, we consider the one-dimensional case. The Gaussian width σ_v of the velocity distribution is associated with the temperature T of the cloud by the relation $\sigma_v^2 = kT/m$, where m stands for the atomic mass and k is the Boltzmann constant. Using Newton's equations for ballistic motion of a particle accelerated by the earth's gravitational field (in the vertical z-direction), the velocity is obtained in terms of the TOF as

$$v_0 = \left(z - z_0 + \frac{1}{2}gt^2\right) \Big/ t, \qquad \frac{\partial v_0}{\partial t} = \frac{\left(z_0 + \frac{1}{2}gt^2 - z\right)}{t^2}. \tag{2}$$

Substituting the above expression for v_0 from equation (2) in equation (1), and then finally integrating over z_0, one can obtain the TOF distribution at an arbitrary distance $z = H$, given by

$$\mathscr{D}(t)\,\mathrm{d}t = \frac{1}{(2\pi t^2)^{1/2}} \frac{\left(\frac{1}{2}gt^2\left(2\sigma_0^2 + \sigma_v^2 t^2\right) - H\sigma_v^2 t^2\right)}{\left(\sigma_0^2 + \sigma_v^2 t^2\right)^{3/2}} \exp\left(-\frac{\left(H + \frac{1}{2}gt^2\right)^2}{2\left(\sigma_0^2 + \sigma_v^2 t^2\right)}\right) \mathrm{d}t. \tag{3}$$

Figure 1 shows a typical TOF or arrival time distribution for cold sodium atoms. These kinds of purely classical analyses are adopted in most of the discussions on TOF measurements where the arrival time of atomic or sub-atomic particles is treated as an elementary well-defined, unique and classical quantity. Also, the theoretical treatments of the TOF distribution that can be obtained using, for instance, Green's function method [4] or any semi-classical method [5], however, are equivalent to the TOF distribution obtained by using Newton's equations for ballistic motion of particles [2]. The interpretations or theoretical analyses of the results of the various TOF experiments [6–8] with molecular, atomic or sub-atomic particles where classical trajectories are inferred from Newtonian mechanics remain debatable, especially in the domain of small atomic masses and low temperatures where quantum mechanical effects should be significant and quantum TOF distribution cannot be reproduced with classical or semi-classical analyses.

Here we provide an example in the context of BEC matter-wave interference, where a quantum analysis for TOF is necessary. We propose a scheme for measuring the TOF distribution of a freely falling atomic BEC prepared in non-classical Schrödinger-cat states. The interference in the TOF distribution (or signal) can then be observed by taking a note or record of the particle counts over various *tiny time windows* at a fixed detector location. This is different from the interference between two freely expanding BECs observed [9] in *space* after a definite time of free fall of the condensates. Coherent splitting of BEC atoms with optically induced Bragg diffraction has been done experimentally [10, 11]. The *spatial coherence* of a BEC is measured using the interference technique by creating and recombining two spatially displaced, coherently diffracted copies of an original BEC [11].

As mentioned above, most of the experiments (particularly when matter waves are associated with center-of-mass motion or external motion of massive quantum particles) demonstrate matter-wave interference by showing the intensity variation at an extended region of detection space at a fixed time. In the present paper, in contrast, we predict and quantify the matter-wave interference in the center-of-mass motion by calculating the *time distribution* of matter-wave arrival probability at some fixed spatial point. More specifically, we discuss here the BEC matter-wave interference in the TOF distribution or *arrival time distribution* since BEC as a source of coherent matter waves is already routinely demonstrated and thus may be an ideal candidate to show an interference signal in the time domain (arrival time distribution). We use here a particular quantum approach to calculate the TOF distribution, and in our analysis we do not use at any point classical or semi-classical ingredients. We consider the free fall of matter wave associated with quantum particles represented by an initial Schrödinger-cat state which is the linear superposition of two mesoscopically distinguishable Gaussian wave packets peaked around different heights, namely, $z = 0$ and $z = -d$, along the vertical z-axis. Then after a certain height of free fall (evolution under the potential $V = mgz$) of the Schrödinger cat, we calculate the quantum TOF distribution at a given detector location $z = H$. During the free fall, the distinct superposed wave packets of the Schrödinger cat overlap or interfere in space, so it is natural to expect that they will also interfere in the *time of fall* showing an interference pattern in the quantum TOF distribution.

We take this particular example of matter-wave interference in the discussion of quantum TOF distribution to pinpoint the necessity of a quantum analysis. So the need for a quantum analysis of TOF distribution is not merely a conceptual issue but a practical one, asking how to predict the TOF distribution using only classical and semi-classical ingredients in a purely quantum scenario like this (interference in the TOF distribution for quantum particles). Now, in spite of the emphasis of quantum theory on the observable concept, there is no commonly accepted recipe to incorporate time observables and their probability distributions in the quantum formalism, and there is considerable difficulty and debate over the issue of defining time (for example, tunneling time, decay time and arrival time) as an observable [12–25]. Even for the simplest case of the arrival time problem there is no unique way to calculate the probability distribution in the quantum formalism [17]. Despite this, many researchers have evidently not been discouraged from seeking an expression for the arrival time distribution (or the quantum TOF distribution) within a consistent theoretical framework. Several logically consistent schemes for the treatment of the arrival time distribution have been formulated, such as those based on axiomatic approaches [14], operator constructions [15], measurementbased approaches [16, 19], trajectory models [21] and the probability current density approach [17–25]. We will use here the probability current density approach to calculate the quantum TOF distribution which is logically consistent and also physically motivating.

The main purpose of our paper is twofold. First, our proposal to experimentally observe or quantitatively predict the matter-wave interference in the time domain, specifically in the TOF distribution is itself quite significant which has not been explored in the current literature to the best of our knowledge. BEC as a source of coherent matter waves is already routinely demonstrated in spatial interference experiments, so BEC will also be an ideal candidate to show the interference in the TOF distribution. Second, we have just mentioned that there is an inherent nonuniqueness within the formalism of quantum mechanics for calculating the TOF or *arrival time* distribution. It remains an open question as to what extent these different quantum mechanical approaches [12–25] for calculating the time distributions can be tested or empirically discriminated. Our proposal of measuring matter-wave interference in TOF distribution has the potential to empirically resolve ambiguities inherent in the theoretical formulations of the quantum TOF distribution. In this respect, it would be interesting if the prediction of BEC matter-wave *interference* in the TOF distribution (calculated from different quantum approaches) be verified in actual experiments.

2. Interference in the quantum time-of-flight distribution for the Bose–Einstein condensate

We begin our analysis with the standard description of the flow of probability in quantum mechanics, which is governed by the continuity equation derived from the Schrödinger equation given by

$$\frac{\partial}{\partial t}|\Psi(\mathbf{x},t)|^2 + \nabla\cdot\mathbf{J}(\mathbf{x},t) = 0. \tag{4}$$

The quantity $\mathbf{J}(\mathbf{x},t) = \frac{i\hbar}{2m}(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi)$ defined as the probability current density corresponds to this flow of probability. In one dimension, the current density $J(x,t)$ tells us the rate at which probability is flowing past the point x. So, interpreting the one-dimensional continuity equation in terms of the flow of physical probability, the Born interpretation for the squared modulus of the wavefunction and its time derivative suggest that the arrival time distribution of the particles reaching a detector located at $x = X$ can be calculated [17–25] using the probability current density $J(x,t)$. It should also be noted that $J(x,t)$ can be negative; hence, one needs to take the modulus sign in order to use the above definition. Our aim here is to derive an expression for the TOF distribution through the quantum probability current density for the atomic BEC representing the mesoscopic Schrödinger cat and showing interference in the TOF signal. The probability current density approach to the TOF distribution is also justified by the Bohmian model of quantum mechanics in terms of the causal trajectories of individual particles [21]. Although the Schrödinger probability current density is formally nonunique up to a total divergence term [20], the current can be uniquely fixed if one calculates the current in the non-relativistic limit of a proper relativistic wave equation which provides appropriate spin-dependent corrections to it [23, 26]. We ignore this small spin-dependent contribution here in our present discussion, as the estimated magnitude of the spin-dependent current is roughly 10^5 to 10^6 times smaller than the Schrödinger current. It was emphasized that the probability current density approach not only provides an unambiguous definition of the arrival time at the quantum mechanical level [17–25], but also addresses the issue of obtaining the proper classical limit of the TOF of massive quantum particles [24, 25].

Now, to keep the discussion concise, here we restrict ourselves to the case of a one-dimensional motion, but our resulting conclusion does not depend on a three-dimensional extension which will be straightforward as discussed later at the end of this section. A magnetically trapped BEC as a source of coherent matter wave or atom laser, where a macroscopic number of atoms occupy the same ground state is now routinely available. After being released from the trapped Bose–Einstein condensates, matter waves fall freely due to the gravity. If the atomic beam is well collimated, we can use a one-dimensional Gross–Pitaevskii equation [3, 27] for the evolution of the condensate wavefunction $\Psi(z, t)$ with the gravitational potential,

$$i\hbar \frac{\partial}{\partial t}\Psi(z,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\Psi(z,t) + mgz\Psi(z,t) + U_0|\Psi(z,t)|^2\Psi(z,t), \tag{5}$$

where $|\Psi(z,t)|^2$ provides the density profile of the BEC, m denotes the atomic mass, g denotes the gravity acceleration and U_0 denotes the inter-atomic interaction strength. In our present discussion we consider the condensate of non-interacting bosons and we neglect [27, 28] the effects of inter-atomic interaction U_0 on the freely falling condensate. In the BEC, the whole complex is described by one single wavefunction $\Psi(z, t)$ (a macroscopic wavefunction of the condensate) exactly as in a single atom, and we can speak of coherent matter in the same way as coherent light in the case of a laser. To show *interference* in the quantum TOF signal for the freely falling BEC, we consider the initial state of the BEC prepared in a Schrödinger-cat state which is the coherent superposition of two mesoscopically distinguishable states in the configuration space:

$$\Psi(z,0) = \mathcal{N}[c_1\psi_1(z,0) + c_2\psi_2(z,0)], \tag{6}$$

where

$$\psi_1(z,0) = \frac{1}{\left(2\pi\sigma_0^2\right)^{1/4}} \exp\left(-\frac{z^2}{4\sigma_0^2}\right), \tag{7}$$

$$\psi_2(z,0) = \frac{1}{\left(2\pi\sigma_0^2\right)^{1/4}} \exp\left(-\frac{(z+d)^2}{4\sigma_0^2}\right) \tag{8}$$

are the Gaussian wave packets centered around $z = 0$ and $z = -d$, respectively, and σ_0 is the initial position spread. A description of the initial 1D wavefunction for two separated BECs using the Gaussian form (6) has been made, for example, by the authors of [29], where they consider all the non-interacting bosons are prepared to be condensed in the ground state of the harmonic trap [3]. For simplicity, we take $c_1 = c_2 = 1/\sqrt{2}$ which implies that after coherent splitting of the original BEC, each component has an equal number of atoms. Then the value of the normalization constant

$$\mathcal{N} = 1\Big/\sqrt{1 + \exp\left(-d^2\big/8\sigma_0^2\right)}. \tag{9}$$

As we have mentioned, the Schrödinger-cat state of matter was generated for a BEC represented by superposition of spatially separated states and the superposition was verified [11] by detecting the quantum mechanical interference (*in space*) between the localized wave packets separated by a mesoscopic distance. Under the experimental situations (where the spatial coherence of the BEC was measured using the interference technique by creating and recombining two spatially displaced, coherently diffracted copies of an original BEC) discussed by the authors of [10, 11], the BEC wavefunction can be written as a linear superposition of spatially separated wave packets [29] which may be inferred as a true macroscopic Schrödinger cat. Now considering the free fall of the coherently split BEC under gravity, we calculate the time evolution of the Schrödinger-cat state (6) according to equation (5) with $U_0 = 0$, we then obtain

$$\Psi(z,t) = \frac{\mathcal{N}}{\sqrt{2}}[\psi_1(z,t) + \psi_2(z,t)], \tag{10}$$

where

$$\psi_1(z,t) = \left(2\pi s_t^2\right)^{-1/4} \exp\left[\frac{-\left(z+\frac{1}{2}gt^2\right)^2}{4s_t\sigma_0}\right] \exp\left[-\mathrm{i}\left(\frac{m}{\hbar}\right)\left(gtz + \frac{1}{6}g^2t^3\right)\right], \tag{11}$$

and

$$\psi_2(z,t) = \left(2\pi s_t^2\right)^{-1/4} \exp\left[\frac{-\left(z+d+\frac{1}{2}gt^2\right)^2}{4s_t\sigma_0}\right] \exp\left[-\mathrm{i}\left(\frac{m}{\hbar}\right)\left(gtz + \frac{1}{6}g^2t^3\right)\right] \tag{12}$$

with

$$s_t = \sigma_0\left(1 + \mathrm{i}\hbar t/2m\sigma_0^2\right). \tag{13}$$

The expression for the Schrödinger probability current density corresponding to the time evolved state $\Psi(z,t)$ (10) is given by

$$J(z,t) = \frac{i\hbar}{2m}\left(\Psi\frac{\partial\Psi^*}{\partial z} - \Psi^*\frac{\partial\Psi}{\partial z}\right) = \frac{\mathcal{N}^2}{2}[J_1(z,t) + J_2(z,t) + J_3(z,t) + J_3{}^*(z,t)], \tag{14}$$

where

$$J_1(z,t) = \left[\frac{\hbar^2 t}{4m^2\sigma_0^2\sigma^2}\left(z + \frac{1}{2}gt^2\right) - gt\right] \times |\psi_1(z,t)|^2, \tag{15}$$

$$J_2(z,t) = \left[\frac{\hbar^2 t}{4m^2\sigma_0^2\sigma^2}\left(z + d + \frac{1}{2}gt^2\right) - gt\right] \times |\psi_2(z,t)|^2, \tag{16}$$

$$J_3(z,t) + J_3{}^*(z,t) = 2P_{12}(z,t)(\eta\cos\delta - \lambda\sin\delta), \tag{17}$$

where

$$\lambda = \frac{\hbar d}{4m\sigma^2}, \tag{18}$$

$$\eta = \frac{\hbar^2 t}{8m^2\sigma_0^2\sigma^2}(2z + d + gt^2) - gt, \tag{19}$$

and

$$P_{12}(z,t) = |\psi_1(z,t)||\psi_2(z,t)| \tag{20}$$

with the time-dependent position spread given by

$$\sigma^2 = s_t s_t^* = \sigma_0^2\left(1 + \hbar^2t^2/4m^2\sigma_0^4\right). \tag{21}$$

Here the quantity s_t is defined in equation (13). The oscillatory factor δ in (17), responsible for the interference effect, is given by

$$\delta = \frac{\hbar t}{8m\sigma_0^2\sigma^2}(d^2 + dgt^2 + 2zd) = \frac{\hbar t(d^2 + \mathrm{dg}\,t^2 + 2zd)}{8m\left(\sigma_0^4 + \frac{\hbar^2t^2}{4m^2}\right)}. \tag{22}$$

Now taking the modulus of the quantum probability current density (14), we obtain the quantum TOF distribution [17–25] at a detector location $z = H$ for the spatially separated BEC Schrödinger cat falling freely under gravity given by

$$\Pi(t) = |J(z = H, t)|. \tag{23}$$

Exactly the same expression for quantum TOF distribution can be obtained for a three-dimensional analysis of the problem by considering the form of the initial wavefunction

$$\Psi(x, y, z, 0) = \frac{\mathcal{N}}{\sqrt{2}} [\psi_1(x, y, z, 0) + \psi_2(x, y, z, 0)], \tag{24}$$

$$\psi_1(x, y, z, 0) = \frac{1}{(2\pi\sigma_0^2)^{3/4}} \exp\left(-\frac{x^2}{4\sigma_0^2}\right) \exp\left(-\frac{y^2}{4\sigma_0^2}\right) \exp\left(-\frac{z^2}{4\sigma_0^2}\right), \tag{25}$$

$$\psi_2(x, y, z, 0) = \frac{1}{(2\pi\sigma_0^2)^{3/4}} \exp\left(-\frac{x^2}{4\sigma_0^2}\right) \exp\left(-\frac{y^2}{4\sigma_0^2}\right) \exp\left(-\frac{(z+d)^2}{4\sigma_0^2}\right), \tag{26}$$

where $\psi_1(x, y, z, 0)$ and $\psi_2(x, y, z, 0)$ are now three-dimensional Gaussian wave packets separated along the vertical $\widehat{\mathbf{z}}$-axis, having peaks around the points $(0, 0, 0)$ and $(0, 0, -d)$, respectively. The value of the normalization constant $\mathcal{N}$ remains the same as that in equation (9). One can then obtain the three-dimensional Schrödinger time evolved wavefunction under the gravitational potential. The quantum TOF distribution can then be calculated again using the three-dimensional quantum current. Interpreting again the three-dimensional continuity equation (4) in terms of the flow of physical probability, one can define the quantum TOF distribution for the atoms crossing a surface element $\mathrm{d}\mathbf{S}$ as $|\mathbf{J}\cdot \mathrm{d}\mathbf{S}|$. It is important to mention here that the quantum flux density $|\mathbf{J}\cdot \mathrm{d}\mathbf{S}|$ has been identified with the 'time distribution' of particles crossing the surface element $\mathrm{d}\mathbf{S}$ by Daumer *et al* [30], who applied Bohm's model to the scattering problem for a quantum particle in three dimensions. Hence, the quantum TOF distribution for atoms reaching a finite surface plane S in three dimensions is given by

$$\mathbf{\Pi}(t) = \left|\int_S\int \mathbf{J}\cdot \mathrm{d}\mathbf{S}\right| = \left|\int_S\int \mathbf{J}\cdot\widehat{\mathbf{n}}\,\mathrm{d}S\right|, \tag{27}$$

where $\widehat{\mathbf{n}}$ is the unit vector normal to the surface. The quantum TOF distribution for the atoms reaching the XY-plane ($\widehat{\mathbf{n}} = -\widehat{\mathbf{z}}$) after a certain height ($z = H$) of free fall is then given by

$$\mathbf{\Pi}_1(t) = \left|\int_S\int \mathbf{J}\cdot\widehat{\mathbf{n}}\,\mathrm{d}S\right| = \left|\int_x\int_y J_z(x, y, z = H, t)\,\mathrm{d}x\,\mathrm{d}y\right|, \tag{28}$$

where $J_z(x, y, z = H, t)$ is the z-component of the three-dimensional probability current density at a fixed height $z = H$. By evaluating the integral of equation (28), one can see that $\mathbf{\Pi}_1(t)$ is exactly the same as the quantum TOF distribution $\Pi(t)$ obtained for one-dimensional analysis (23). To understand more clearly the origin of this interference in TOF, let us consider the propagation (evolution) of individual wave packets ψ_1 and ψ_2 under the gravitational potential. Then one will have two distinct TOF distributions having separate mean arrival times. This is because the peaks of the component wave packets ψ_1 and ψ_2 take different times to reach the detector at $z = H$, since they are spatially separated along the vertical z-axis, and the interference in TOF arises due to the superposition of these two wave packets. In this setup, the cross term (interfering term) in the quantum TOF distribution arises from the relative phase of the component wave packets (ψ_1 and ψ_2) along the z-direction, as only z-components of the component wave packets differ in the time evolution and continue to develop the relative phase, and this relative phase is not cancelled out when we perform the integration over the *XY*-plane. The interference pattern in the quantum TOF distribution can be detected by using a probe laser, focused in the form of a sheet underneath the falling BEC atoms in the *XY*-plane at $z = H$. When the trapping forces are turned off, the BEC atoms will fall through the laser probe under the influence of gravity. It is then possible to detect the fluorescence from the atoms excited by the resonant probe light as they reach the detection sheet. The fluorescence can be measured as a function of time to determine the TOF distribution. For this setup, one can also consider a situation where the detection is made in the *YZ*-plane ($\widehat{\mathbf{n}} = -\widehat{\mathbf{x}}$) at a fixed $x = X$. In that case, quantum TOF distribution (say, $\mathbf{\Pi}_2(t)$) can be obtained from the x-component of the three-dimensional current ($J_x(x = X, y, z, t)$) integrated over the *YZ*-plane using equation (27). By evaluating that integral, one can see that there will be no interference in the quantum TOF distribution $\mathbf{\Pi}_2(t)$ under this situation. This is because the interference term in the quantum TOF distribution is wiped out when we perform the integration over the *YZ*-plane.

Next, we consider another setup in three dimensions where the superposed wave packets $\psi_1(x, y, z, 0)$ and $\psi_2(x, y, z, 0)$ are separated along the horizontal X-axis, having peaks around the points (0, 0, 0) and $(-d, 0, 0)$, respectively. This situation is analogous to the experimental setup of [9] where BEC interference was observed in *space* at a fixed time. For this geometry, one can again calculate the three-dimensional Schrödinger time evolved wavefunction under the gravitational potential. The quantum TOF distribution (27) can then be calculated again using the three-dimensional quantum current. For this setup, we again consider the detection of the particles at a surface plane (*XY*-plane with $\widehat{\mathbf{n}} = -\widehat{\mathbf{z}}$) at $z = H$. In this case, quantum TOF distribution (say, $\mathbf{\Pi}_3(t)$) can be obtained from the z-component of the three-dimensional current integrated over the *XY*-plane using equation (27). By evaluating that integral, one can see that there will not be any interference at all in the quantum TOF distribution $\mathbf{\Pi}_3(t)$. The interference term in the quantum TOF distribution is wiped out when we perform the integration over the *XY*-plane. This is because for this setup, the individual wave packets ψ_1 and ψ_2 are not separated along the vertical z-axis, so they will have the same TOF distribution with the same mean arrival time to reach the detection plane at $z = H$. Hence, we do not expect any interference in the quantum TOF distribution detected at the horizontal plane at $z = H$ when we consider the superposition of the horizontally separated wave packets. The interference term in the quantum TOF distribution $\mathbf{\Pi}_3(t)$, in this case, is wiped out when we perform the integration over the *XY*-plane, even though one can observe the interference in space at a fixed time. For this setup, one can also consider a situation where the detection is made in the *YZ*-plane ($\widehat{\mathbf{n}} = -\widehat{\mathbf{x}}$) at a fixed $x = X$. In that case, quantum TOF distribution (say, $\mathbf{\Pi}_4(t)$) can be obtained from the x-component of the three-dimensional current integrated over the *YZ*-plane using equation (27). For this situation, although we will see the presence of some interfering terms in the expression of quantum TOF distribution $\mathbf{\Pi}_4(t)$, the intensity to observe this interference will be very low, as only a small fraction of the condensate atoms will arrive at the detection *YZ*-plane at $x = X$ due to free expansion (free particle motion) of the wave packets. One can also check that the quantum TOF distribution $\mathbf{\Pi}_4(t)$ will have exactly the same expression as $\Pi(t)$ of (23) with $g = 0$ (no gravity) and with $z = H$ replaced by $x = X$. We will show numerically in the next section that gravity plays an important role in our setup to pull down the vertically separated superposed condensate toward the detection plane at $z = H$.

Hence, the only two situations (in our above discussion) where we see the presence of interference in three dimensions are the quantum TOF distribution $\mathbf{\Pi}_1(t)$ and $\mathbf{\Pi}_4(t)$. Now, $\mathbf{\Pi}_1(t)$ is exactly the same as $\Pi(t)$ and $\mathbf{\Pi}_4(t)$ is also the same as $\Pi(t)$ with $g = 0$ and with

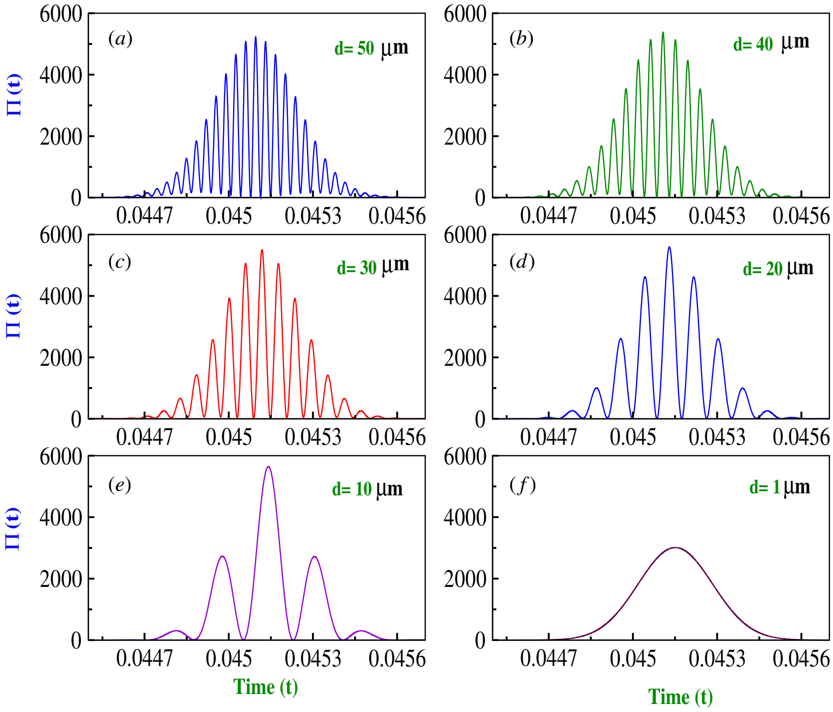

Figure 2. Quantum TOF distributions $\Pi(t)$ for the coherently splitted BEC of sodium atoms (representing the macroscopic Schrödinger cat) falling freely under gravity are plotted for varying wave-packet separation d. In each curve, time (in s) is plotted along the horizontal direction and the coherent TOF distribution $\Pi(t)$ of the BEC Schrödinger cat is plotted (in s^{-1}) along the vertical axis. The detector is located at a distance $z = H = -1$ cm and $\sigma_0 = 1$ μm.

$z = H$ replaced by $x = X$. So the whole characteristic of the interference pattern in quantum TOF distribution hinges upon the form of $\Pi(t)$. In the next section we study numerically the parameter dependence of the quantum TOF distribution $\Pi(t)$ and the physical interplay between these parameters.

3. Numerical results and discussions

Quantum TOF distribution $\Pi(t)$ of the freely falling atomic BEC Schrödinger cat is plotted (figure 2) at a detector location $z = H = -1$ cm with different values of wave-packet separation d. We see clear signature of interference in the quantum TOF distribution arising due to the terms $J_3(z = H, t)$ and $J_3{}^*(z = H, t)$ of (17) in the expression for quantum probability current density (23) and (14). During free fall, the spatially separated wave packets of the BEC Schrödinger cat overlap or interfere in space and hence they also interfere in the time of fall showing an interference pattern in the quantum TOF distribution. The quantum TOF distribution $\Pi(t)$ may be visualized as a coherent pulse of BEC atoms. The interference pattern in the quantum TOF signal (figure 2(a)) is very sharp for a typical set of parameter values, for example, $H = -1$ cm, $d = 50$ μm, $\sigma_0 = 1$ μm, and the pattern disappears (figure 2(f)) when the separation between the BEC superposed wave packets is decreased to $d = 1$ μm for the above-mentioned parameter values. We can see from the

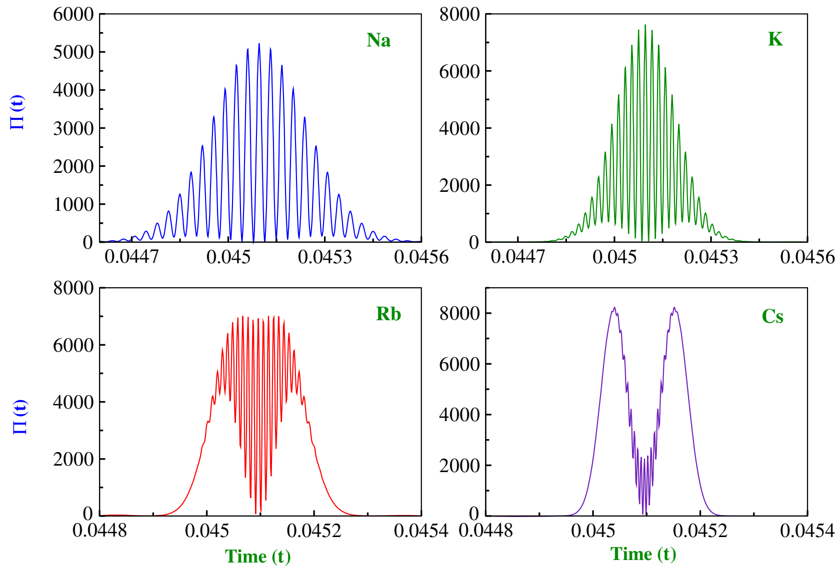

Figure 3. Quantum TOF distributions $\Pi(t)$ for the coherently splitted BEC of sodium atoms (representing the macroscopic Schrödinger cat) falling freely under gravity are plotted for varying atomic masses. In each curve, time (in s) is plotted along the horizontal direction and the TOF signal $\Pi(t)$ is plotted (in s^{-1}) along the vertical axis. The detector is located at $z = H = -1$ cm with $d = 50$ μm and $\sigma_0 = 1$ μm.

oscillatory factor δ (22) in (17) that the number of oscillations and hence the number of fringes increases in the TOF distribution $\Pi(t)$ (23) as one increases the separation d. The interference effect arises mainly because of two factors: one is the temporal overlap $P_{12}(z = H, t)$ (20) and the other is the oscillatory factor δ. When d is very small, the overlap $P_{12}(z = H, t)$ is very high, but the oscillatory factor δ becomes small. As a consequence, the oscillation frequency is too slow or the oscillation period is too large, and we do not see any oscillatory effect in the temporal overlap region of the wave packets. The number of oscillations increases as one increases d, but again after a certain value ($d > 400$ μm) of separation there will be no interference as the overlap $P_{12}(z = H, t)$ becomes very small in that case.

From figure 3, we see that the interference pattern in the quantum TOF signal gradually disappears as one increases the mass m of the atoms. Figure 4 shows the quantum TOF distribution for different values of wave-packet width from $\sigma_0 = 1$ μm to $\sigma_0 = 6$ μm. It is clear from figure 4 that the number of fringes and the contrast of interference patterns in quantum TOF distribution decreases as one increases the value of the initial widths (σ_0) of the wave packets. Nevertheless, it is possible to see the interference for a larger value of σ_0. For example, if one chooses $\sigma_0 = 10$ μm, then to observe a good interference pattern (with good contrast and having a considerable number of fringes) in $\Pi(t)$, the separation d needs to be considered in the range of 50 μm to 250 μm, with the detector placed at a longer distance ($H = -100$ cm) for a fixed mass of sodium atoms.

The interference in $\Pi(t)$ is sensitive to the parameters σ_0 and the atomic mass m, the detector location H and the separation d. We repeat here that the interference in $\Pi(t)$ arises mainly because of the temporal overlap $P_{12}(z = H, t)$ (20) and the oscillatory factor δ (22). To increase the temporal overlap $P_{12}(z = H, t)$, one has to find the condition under which the spreading of the wave packet increases: small σ_0, lighter mass atoms and a distant detector

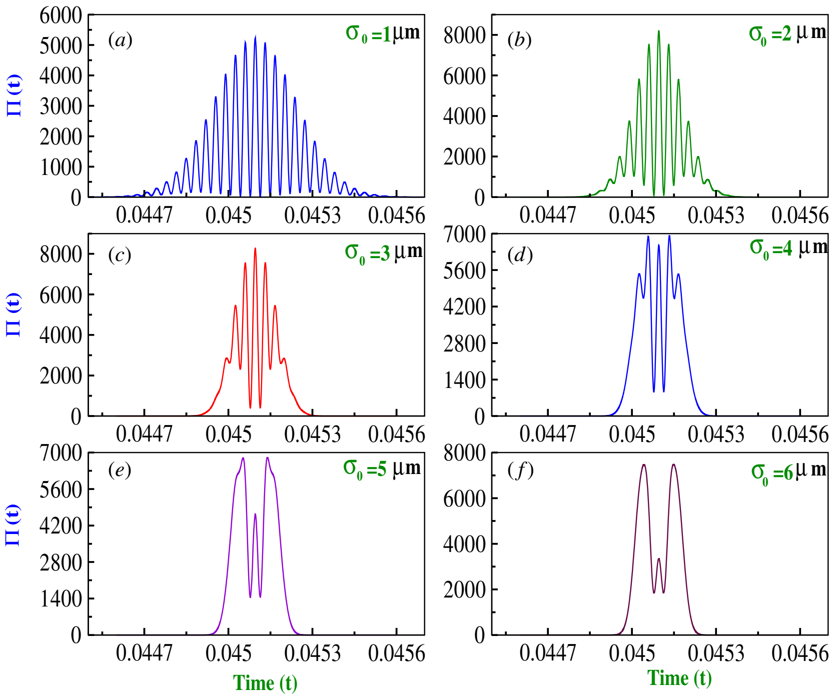

Figure 4. Quantum TOF distributions $\Pi(t)$ for the coherently splitted BEC of sodium atoms (representing the macroscopic Schrödinger cat) falling freely under gravity are plotted for varying wave packet width σ_0. In each curve, time (in s) is plotted along the horizontal direction and the coherent TOF distribution $\Pi(t)$ of the BEC Schrödinger cat is plotted (in s^{-1}) along the vertical axis. The detector is located at a distance $z = H = -1$ cm and $d = 50$ μm.

location (large H) will be helpful in this regard to enhance this effect. The oscillatory factor δ can be increased either by reducing the value of σ_0, or by increasing the parameters d and H. Actually, when one considers higher values of the parameter σ_0, then the temporal overlap $P_{12}(z = H, t)$ and the oscillatory factor δ both decrease. This is because for larger values of the parameters σ_0 (or mass m), the spreading effect (21) and hence the temporal overlap $P_{12}(z = H, t)$ becomes small. As a result, the wave packets try to localize (in time as well as in space) more strongly causing the interference effect to be small. Also, for higher values of σ_0, the oscillatory factor δ will be too small due to the presence of σ_0^4 in the denominator of δ (22). Then one has to allow the BEC to travel a longer distance (by increasing H) to develop some temporal overlap of the wave packets, and also increasing H helps us to increase δ (22). For higher values of σ_0, the parameter δ should also be increased by increasing the value of the separation d, keeping in mind that there remains a considerable temporal overlap $P_{12}(z = H, t)$. The temporal overlap gets reduced if one increases the separation d too much. So, even if there is a delicate choice of the parameters, one can observe the TOF interference for a wide range of parameter values.

It is significant to mention here that gravity plays an important role in our setup to observe the interference in the TOF distribution. In figure 5 we plot the time distribution $\Pi(t)$ for $g = 0$ (no gravity) and compare it with figure 2(d) where we plot $\Pi(t)$ in the presence of gravity with the parameter values the same as that of figure 5. We see that the magnitude of $\Pi(t)$ (in the absence of gravity) is roughly 10^5 times smaller than that obtained for the

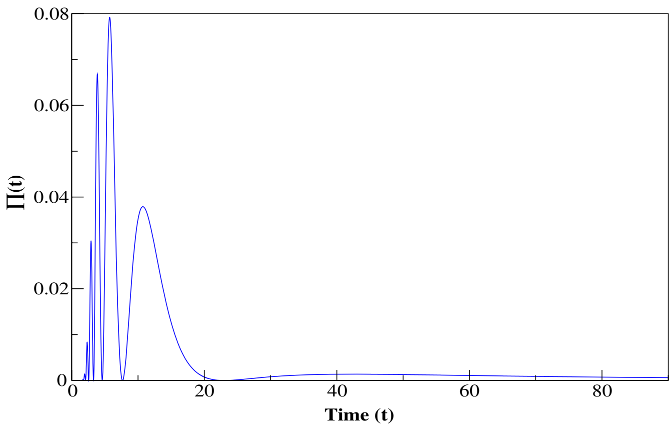

Figure 5. Quantum TOF distribution $\Pi(t)$ for the coherently splitted BEC of sodium atoms (representing the macroscopic Schrödinger cat) is plotted in the absence of gravity ($g = 0$). Time (in s) is plotted along the horizontal direction and the coherent TOF distribution $\Pi(t)$ of the BEC Schrödinger cat is plotted (in s^{-1}) along the vertical axis. The detector is located at a distance $z = H = -1$ cm with $\sigma_0 = 1$ μm and $d = 20$ μm.

gravitational free fall case for $z = H = -1$ cm. The reason for this is that the magnitudes of $J_1(z = H, t)$ and $J_2(z = H, t)$ become very small (roughly 10^5 times) in the absence of gravity. This magnitude becomes 10^6 times smaller if we consider the detector location at $z = -10$ cm. Actually, in the absence of gravity, there will be free particle motion and expansion of the wave packets in every direction. So, if one tries to observe the interference in the quantum TOF distribution in the absence of gravity, the intensity of that interference pattern will be too faint to observe as only a small fraction of the condensate atoms will arrive at the detector. Hence, in our setup, gravity plays an important role which helps to pull down the condensate towards the detection plane.

4. Summary and conclusion

To summarize, in this work we propose a scheme to experimentally observe matter-wave interference in the time domain, specifically in the TOF (arrival time) distribution using atomic BEC. This experimentally testable scheme has the potential to empirically resolve ambiguities inherent in the theoretical formulations of the quantum arrival time distribution. Here we use the probability current density approach to calculate the quantum TOF distributions for an atomic BEC Schrödinger cat represented by superposition of macroscopically separated wave packets in space. Our definition of the quantum TOF distribution in terms of the modulus of the probability current density is particularly motivated from the equation of continuity, and other physical considerations discussed in the literature [17–25]. This approach also provides a proper classical limit, as the interference and hence the coherence in the quantum TOF signal disappears in the large-mass limit. We repeat that there is no classical analogue of this TOF distribution $\Pi(t)$ and this is purely a quantum distribution where we see the matter-wave interference in the quantum TOF signal. Hence, it will be interesting to see if our prediction of interference in the time domain (TOF distribution) can be verified in actual experiments using modern interferometry techniques and sophisticated TOF methods.

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